

Markscheme

May 2023

Extended Mathematics

On-screen examination

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The markscheme abbreviations:

- Bullet notation means award 1 mark – see example below

Example 1
.1 mark awarded and corresponding notes are aligned

b	.1 Show clear line of reasoning in the method	.1 45 and 49 seen OE	2
	.2 4	<i>Ex:</i> $49 = 45 + x$.2	

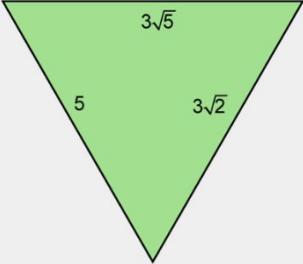
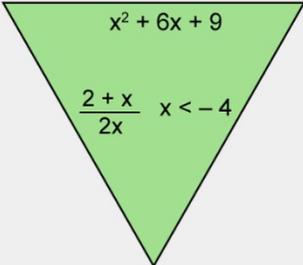
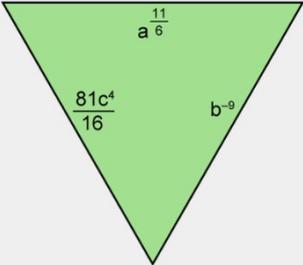
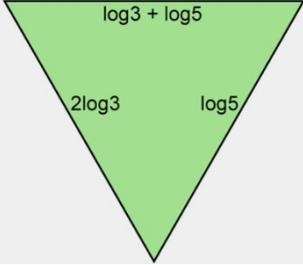
Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- To award **ECF** marks for a question part, **there must be working present for that part**.
- ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- A misread (**MR**) is an error. **ECF** is normally awarded.

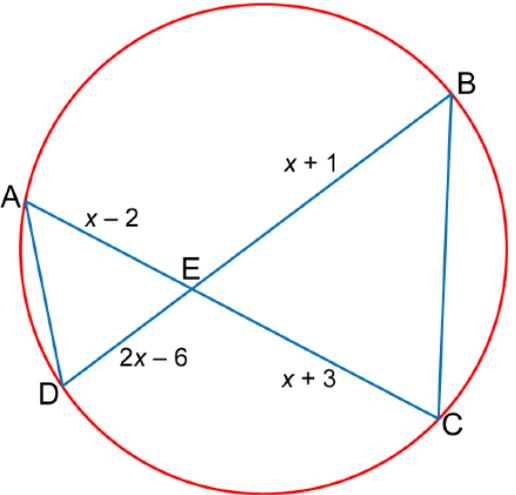
General points

- As this is an international examination, accept all **alternative forms of notation**, for example 1,9 as 1.9 ; 1,000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation x^2 for x^2 unless noted otherwise in the MS.
- ACCEPT** notation errors in intermediate steps.
- Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradict the correct answer**, then that last mark cannot be awarded.
- In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- Where candidates have written two solutions to a question, mark the first solution.
- In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. $\frac{1}{2}$ **OR** 1/2 **OR** 0.5 **OR** $2 \div 4$; $\frac{x}{2}$ **OR** $x / 2$ or $x \div 2$; 0.23 **OR** 23%
- In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks. However, it indicates what the candidate's result represents. Ex: if last mark is for the result: $(AB)=5$; this means we award the mark for seeing 5 as the result of calculating AB without necessarily seeing $AB=5$, but it does not mean we award the mark for seeing 5 representing another length
- Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- Accept seeing equation not in-line.
- Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- ACCEPT** using the correct values or working regardless their previous result.
- Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. **If in doubt, contact your team leader for advice.**
- Unless noted otherwise, if a note in a part says to accept the answer without working for 1 mark less than total marks, then seeing the correct answer with any acceptable working step, award full marks. Example: If the note in a part worth 3 marks says "4.3(3...) without working award 2 marks", then seeing any acceptable working step and seeing 4.3(3...) as the answer award the 3 marks.
- For "**show that**" questions, unless otherwise noted, every bullet point has to be seen in order to be awarded.

Q1	Answers	Notes	Total
a	Place the correct triangle		1
b	Place the correct triangle		1
c	Place the correct triangle		1
d	Place the correct triangle		1

Q2	Answers	Notes	Total
a	.1 Three correct numerators .2 All correct numerators	<p> A probability tree diagram showing three stages of drawing. Stage 1: - D: $\frac{10}{25}$ - W: $\frac{15}{25}$ Stage 2 (from D): - D: $\frac{9}{24}$ - W: $\frac{15}{24}$ Stage 2 (from W): - D: $\frac{10}{24}$ - W: $\frac{14}{24}$ Stage 3 (from D, D): - D: $\frac{8}{23}$ - W: $\frac{15}{23}$ Stage 3 (from D, W): - D: $\frac{9}{23}$ - W: $\frac{14}{23}$ Stage 3 (from W, D): - D: $\frac{9}{23}$ - W: $\frac{14}{23}$ Stage 3 (from W, W): - D: $\frac{10}{23}$ - W: $\frac{13}{23}$ </p>	2

	<p>b</p>	<p>.1 One correct product of three probabilities (DDW or DWD or WDD)</p> <p>.2 The other two correct products of three probabilities OR 3 × their .1</p> <p>.3 Correctly add their three probabilities OR their correct probability after multiplying by 3</p>	<p>.1 $\frac{10}{25} \times \frac{9}{24} \times \text{their } \frac{15}{23}$ OR $\frac{10}{25} \times \text{their } \frac{15}{24} \times \frac{9}{23}$ OR $\frac{15}{25} \times \frac{10}{24} \times \frac{9}{23}$ OR $\frac{9}{92}$ OR 0.0978(3...) ACCEPT 0.098 or 0.1</p> <p>.2 $\frac{10}{25} \times \frac{9}{24} \times \text{their } \frac{15}{23}$ AND $\frac{10}{25} \times \text{their } \frac{15}{24} \times \frac{9}{23}$ AND $\frac{15}{25} \times \frac{10}{24} \times \frac{9}{23}$ OR $3 \times \text{their } \frac{9}{92}$ OR $3 \times 0.0978(3...)$</p> <p>.3 their $\frac{27}{92}$ or 0.29(3478...) OE ACCEPT 27/92</p> <p>.3 DO NOT ACCEPT if they add more than three .3 ONLY ACCEPT if their probability is less than 1</p>	<p>3</p>
	<p>c</p>	<p>.1 Correctly substitute $P(A \cap B)$ and their part b) into the correct conditional probability formula</p> <p>.2 Correctly determine their $P(A B)$</p>	<p>.1 $\frac{15}{25} \times \frac{10}{24} \times \frac{9}{23}$ or $\frac{0.0978(260...)}{0.29(3478...)}$ or $\frac{9}{92}$ or $\frac{0.0978(260...)}{\text{their part b)}$ ACCEPT $\frac{0.1}{\text{their part b)}$</p> <p>.1 DO NOT ACCEPT $P(A \cap B)$ being $P(A) \times \text{their } P(B)$</p> <p>.2 their $\frac{1}{3}$</p> <p>.2 ACCEPT only if their $\frac{1}{3}$ is less than 1</p>	<p>2</p>
	<p>d</p>	<p>.1 Correctly multiply $P(A)$ and their $P(B)$ from part b)</p> <p>.2 Correctly state their .1 does not equal $P(A \cap B)$</p> <p>Event A and B are not independent AG</p>	<p>.1 $(\frac{15}{25} \times \text{their } \frac{27}{92} =)$ their $\frac{81}{460}$ or their 0.176(08695...) ACCEPT 0.18</p> <p>.1 DO NOT ACCEPT if they don't have a calculated product</p> <p>.2 Their .1 does not equal to their $\frac{9}{92}$ WTTE</p> <p>.2 DO NOT ACCEPT if they don't have a calculated product from .1</p> <p>DO NOT ACCEPT any explanation about independency of events even though it seems correct</p>	<p>2</p>

Q3	Answers	Notes	Total
a	<p>AM1 .1 Recognize a first pair of congruent angles (with or without reason) .2 Recognize a second pair of congruent angles (with or without reason)</p> <p>AM2 .1 Correctly apply chords inside circle theorem to recognize that sides are in the same proportion. .2 Recognize vertically opposite angles</p> 	<p>AM1 .1 One from the list of possibilities .2 Another from the list of possibilities</p> <p>List of possibilities : - inscribed angles subtended by the same arc $\angle ADE = \angle BCE$ or $\angle DAE = \angle CBE$ - the vertically opposite angles $\angle AED = \angle BEC$</p> <p>AM2 .1 $\overline{BE} \times \overline{ED} = \overline{AE} \times \overline{EC}$ OE AND $\frac{\overline{BE}}{\overline{EC}} = \frac{\overline{AE}}{\overline{ED}}$ or $\frac{\overline{BE}}{\overline{AE}} = \frac{\overline{CE}}{\overline{DE}}$ OE .2 $\angle AED = \angle BEC$</p> <p>ACCEPT correct angles using different labels CAD or A for $\angle EAD$ DAC or A for $\angle DAE$ DBC or B for $\angle EBC$ CBD or B for $\angle CBE$ ACB or C for $\angle ECB$ BCA or C for $\angle BCE$ ADB or D for $\angle ADE$ BDA or D for $\angle EDA$</p> <p>ACCEPT saying equal WTTE ex: congruent, the same, etc</p> <p>DO NOT ACCEPT E for any angle around E DO NOT ACCEPT $\angle E = \angle E$ or lines create two congruent angles at E DO NOT ACCEPT AC and BD are chords on the circumference DO NOT ACCEPT AED similar to BEC</p>	2

	c	<p>.1 Correctly substitute using $x=5$ into cosine rule</p> <p>.2 Correctly calculate their BC^2 after substituting into cosine rule</p> <p>.3 Correctly calculate their BC from their BC^2</p> <p>.4 Their correct values of m and n</p>	<p>.1 $(BC^2 =) 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 60$ OE</p> <p>.2 $(BC=)$ their 52</p> <p>.3 their $\sqrt{52}$ ACCEPT decimals their 7.2(111)</p> <p>.3 Seeing $\sqrt{52}$ or 7.2(111) implies .2</p> <p>.4 $m=2$ and $n = 13$ ACCEPT seeing $2\sqrt{13}$</p> <p>.4 ACCEPT $m=1$ and $n=$their52 only if their52 is an integer</p> <p>.4 ACCEPT their correct m and n from their 52</p>	4
	d	<p>.1 Correct ratio</p> <p>.2 Correctly apply their ratio to their $2\sqrt{13}$ to find AD</p> <p>.3 Their perimeter correct after adding 21 to their $2\sqrt{13}$ and their $\sqrt{13}$ in surd form</p>	<p>.1 0.5 or 2 OE seen</p> <p>.1 ACCEPT seeing $2 \times$ their $2\sqrt{13}$</p> <p>.2 their $\sqrt{13}$.2 implies .1</p> <p>.2 ACCEPT in decimals i.e. their 3.6(0555...)</p> <p>.2 DO NOT ACCEPT if their AD = their BC</p> <p>.3 $21 +$ their $3\sqrt{13}$</p> <p>.3 DO NOT ACCEPT decimals</p> <p>.3 DO NOT ACCEPT if answer does not include surd</p> <p>.3 DO NOT ACCEPT the use of "root" in words.</p> <p>.3 DO NOT ACCEPT a perimeter that does not include their AD</p>	3

Q4	Answers	Notes	Total
a	<p>AM1 (Using slant height) .1 Correctly substitute into Pythagoras OR correctly substitute into correct trig ratio OR correctly use area of triangle formula to find slant height</p> <p>.2 Correct value for slant height</p> <p>.3 Correctly substitute their .2 into Pythagoras to find h</p> <p>.4 Correct value of h before simplifying</p> <p>$12\sqrt{2}$ AG</p> <p>AM2 (Using square base) .1 Correctly substitute into Pythagoras OR correctly use trig ratio to find diagonal of the base or half diagonal of the base</p> <p>.2 Correct value for diagonal of the base or half diagonal of the base</p> <p>.3 Correctly substitute half of base into Pythagoras to find h</p> <p>.4 Correct value of h before simplifying</p> <p>$12\sqrt{2}$ AG</p>	<p>AM1</p> <p>.1 $\sqrt{24^2 - 12^2}$ OR $12 \times \tan 60$ OE OR $(\frac{1}{2} \times 24 \times 24 \times \sin 60) \div 12$ OE ACCEPT $24^2 = 12^2 + L^2$ OE or $(h^2 =)24^2 - 12^2$</p> <p>.2 $\sqrt{432}$ or $12\sqrt{3}$ or 20.78(4609...) OE</p> <p>.3 $\sqrt{(\text{their}\sqrt{432})^2 - 12^2}$ or $\sqrt{(\text{their}(12\sqrt{3}))^2 - 12^2}$ or $h^2 + 12^2 = \text{their}432$ OE or their $20.78(4609)^2 - 12^2$ OE</p> <p>.4 $\sqrt{288}$</p> <p>.4 ACCEPT 16.97 AND $12\sqrt{2} = 16.97$</p> <p>AM2</p> <p>.1 $\sqrt{24^2 + 24^2}$ OE OR $\frac{24}{\sin 45}$ OE ACCEPT $24^2 + 24^2$ or $12^2 + 12^2$ OE</p> <p>.2 $\sqrt{1152}$ or $24\sqrt{2}$ or 33.9(4111) OE .2 ACCEPT 16.97 or $12\sqrt{2}$</p> <p>.3 $\sqrt{24^2 - (12\sqrt{2})^2}$ OE or $h^2 + (12\sqrt{2})^2 = 24^2$ OE .3 ACCEPT in decimals .3 DO NOT ACCEPT trig ratio using angle 45</p> <p>.4 $\sqrt{288}$</p> <p>.4 ACCEPT 16.97 AND $12\sqrt{2} = 16.97$.4 ACCEPT only if .2 and .3 are awarded Reaching $\sqrt{288}$ coming from half of 576, award 0 marks See notes on next page</p>	4

		<p>Seeing pyramid made with equilateral triangles so height is half of side $\sqrt{2}$, award 0 marks</p> <p>Calculating the height using the volume of part b, award 0 marks.</p>	
b	<p>.1 Correctly substitute $12\sqrt{2}$ and 24 into volume of pyramid formula</p> <p>.2 Correctly calculate their result after substituting into their volume formula</p> <p>.3 Correctly rounded their volume to the nearest integer</p>	<p>.1 $\frac{1}{3} \times 24 \times 24 \times 12\sqrt{2}$ ACCEPT $\frac{1}{3} \times 24 \times 24 \times 16.97$</p> <p>.1 ACCEPT $\frac{1}{3} \times 576 \times 12\sqrt{2}$</p> <p>.2 their 3258.34(80...)</p> <p>.2 ACCEPT incorrect volume formula providing it includes $24 \times 24 \times 12\sqrt{2}$</p> <p>.2 DO NOT ACCEPT their result if it comes from $\frac{1}{3} \times \text{length} \times 12\sqrt{2}$</p> <p>.3 their 3258</p>	3
c	<p>.1 Correctly calculate the modified ratio for the height</p> <p>.2 The correct value of h</p>	<p>.1 $\sqrt[3]{0.75}$ or 0.9(0856...) OE seen</p> <p>ACCEPT $\frac{1}{\sqrt[3]{0.75}}$ or 1.1 (the inverse ratio)</p> <p>ACCEPT $0.75 = \left(\frac{h}{12\sqrt{2}}\right)^3$ OE</p> <p>.2 ($\sqrt[3]{0.75} \times 12\sqrt{2} =$)15.4(187...)</p>	2

<p>c</p>	<p>.1 Correctly write two values .2 Correctly write a third values .3 Correctly write a fourth value .4 Correctly write fifth and sixth values</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 5%;"></th> <th style="width: 15%;"></th> <th style="width: 15%;">Weight, tonnes (t)</th> <th style="width: 15%;">Production ratio Energy per tonne (TJ / t)</th> <th style="width: 15%;">Energy Tera-joules (TJ)</th> </tr> </thead> <tbody> <tr> <td rowspan="3" style="writing-mode: vertical-rl; transform: rotate(180deg); text-align: center;">Category</td> <td>Domestic</td> <td style="text-align: center;">1350</td> <td>$r =$ 0.1 or 1/10 or 1:10 or "1 to 10"</td> <td style="text-align: center;">135</td> </tr> <tr> <td>Commercial</td> <td>Answer from (a) their 720</td> <td>$r =$ their 0.1 (above)</td> <td>(0.1 × their720=) their72</td> </tr> <tr> <td>Industrial</td> <td>Answer from (b) their 270</td> <td>$2r =$ (2 × their 0.1 =) their 0.2 OE</td> <td>(0.2 × their270=) their54</td> </tr> </tbody> </table> <p style="text-align: right; margin-top: 10px;">Total energy produced by the organization this months (135 + their72 + their54=) their261</p> <p>Answers from (a) and (b) are not counted as one of the six values <u>The six values are:</u> r domestic=0.1 OE ACCEPT 1/10 or "1:10" or "1 to 10" OE r commercial= their r for domestic 2r=(2xtheir r domestic=)their0.2 ACCEPT their2/10 or their "2:10" or their"2 to 10" OE</p> <p>Energy commercial=(0.1 × their720=) their72 Energy industrial=(0.2 × their270=) their54 Total Energy=(135+their72+their54=) their261</p> <p>ACCEPT Energy Commercial 72 or Energy industrial 54 regardless their r value</p>			Weight, tonnes (t)	Production ratio Energy per tonne (TJ / t)	Energy Tera-joules (TJ)	Category	Domestic	1350	$r =$ 0.1 or 1/10 or 1:10 or "1 to 10"	135	Commercial	Answer from (a) their 720	$r =$ their 0.1 (above)	(0.1 × their720=) their72	Industrial	Answer from (b) their 270	$2r =$ (2 × their 0.1 =) their 0.2 OE	(0.2 × their270=) their54	<p>4</p>
		Weight, tonnes (t)	Production ratio Energy per tonne (TJ / t)	Energy Tera-joules (TJ)																	
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	Commercial	Answer from (a) their 720	$r =$ their 0.1 (above)	(0.1 × their720=) their72																	
	Industrial	Answer from (b) their 270	$2r =$ (2 × their 0.1 =) their 0.2 OE	(0.2 × their270=) their54																	
<p>d</p>	<p>.1 Correct mid-interval values .2 Add the product of their mid-interval values by frequency .3 Divide their sum by 12 .4 Correct mean before rounding AG 900</p>	<p>.1 860, 880, 900,920, 940 ACCEPT seeing only three correct</p> <p>.2 $2 \times \text{their860} + 4 \times \text{their880} + 2 \times \text{their900} + 1 \times \text{their920} + 3 \times \text{their940}$ OE or 10780</p> <p>.2 ACCEPT seeing only three correct products OE added</p> <p>.3 Ex: $\frac{\text{their bp2}}{2 + 4 + 2 + 1 + 3}$</p> <p>.4 898(.333...)</p>	<p>4</p>																		

Q6	Answers	Notes	Total
a	<p>.1 Correctly substitute (0,120) into the equation</p> <p>.2 Correct value of a as a product after the substitution of (0,120)</p> <p>$a = 330$ AG</p>	<p>.1 $120 = \frac{a}{(0+6)} + 65$ or $120 = \frac{a}{6} + 65$</p> <p>ACCEPT substitution of other points (n,E) from the graph in .1</p> <p>Accepted points (n,E) from the graph: (0,120), (1,112), (2,106), (3,102), (4,98), (5,95), (6, 93), (7,90), (8, 89) ACCEPT E being ± 5 the values indicated above</p> <p>.2 $(a =) 55 \times 6$</p> <p>.2 ACCEPT $55 = \frac{a}{6}$</p>	2
b	<p>.1 Correctly substitute 2 into the equation</p> <p>.2 Correct answer after their substitution</p>	<p>.1 $E = \frac{330}{(2+6)} + 65$ or $E = \frac{330}{8} + 65$</p> <p>.2 their106.25 OE</p> <p>.2 ACCEPT 106.3 or 106 or 107</p>	2
c	<p>.1 Correctly subtract their result in b) from 120</p> <p>.2 Correctly multiply their13.75 by 60 000</p>	<p>.1 (their120 – their part b) =) their13.75</p> <p>.1 ACCEPT their part b) being correctly rounded up or down as an integer Ex: (120-their106=) their 14 or (120-their107=) their13</p> <p>.2 (their13.75 \times 60000 =) their825 000</p> <p>825 000 without working, award 2 marks (their part b) \times 60000 or their6 375 000, award 1 mark 120 \times 60000 or 7 200 000, award 0 marks</p>	2

	<p>d</p> <p>AM1 (Solving)</p> <p>.1 Correctly equate to 95</p> <p>.2 Correctly rearrange for their n</p> <p>.3 The correct value of n</p> <p>AM2 (using numbers)</p> <p>.1 Correctly substitute one number $n \neq 5$ and correctly calculate corresponding E</p> <p>.2 Correctly substitute 5 and equate to 95</p> <p>.3 Correctly identify the value of n</p>	<p>AM1 (Solving)</p> <p>.1 $95 = \frac{330}{(n+6)} + 65$ or $30 = \frac{330}{(n+6)}$ or $30(n+6) = 330$ OE</p> <p>.1 ACCEPT the use of inequalities</p> <p>.2 $n = \frac{330}{\text{their30}} - 6$ OE</p> <p>.2 ACCEPT $(n+6) = \frac{330}{\text{their30}}$</p> <p>.2 ACCEPT $\text{their30}n = \text{their150}$</p> <p>.2 implies .1</p> <p>.3 $(n=)5$</p> <p>.3 ACCEPT 2027</p> <p>AM2 (using numbers)</p> <p>.1 Ex: $\frac{330}{(1+6)} + 65 = 112.14$ ACCEPT 112</p> <p>.2 $\frac{330}{(5+6)} + 65 = 95$</p> <p>.3 $(n=) 5$ ACCEPT 2027</p>	<p>3</p>
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Q6e total 10 marks

Mark	1	2		
Factors (F)	<p>Two from the keywords/phrases below mentioned in the factors box</p> <ul style="list-style-type: none"> • filter(s) • carbon or unit(s) or kt or emission(s) • limit(s) or restriction(s) or requirement(s) or regulation(s) or “difference in emission(s)” or “value(s) of difference” <p>DO NOT ACCEPT other similar keywords Ex: “excess” or “reduction”</p> <p>ACCEPT multiple keywords in one sentence and ignore additional irrelevant factors</p>	<p>The three keywords/phrases below mentioned in the factors box</p> <ul style="list-style-type: none"> • filter(s) • carbon or unit(s) or kt or emission(s) • limit(s) or restriction(s) or requirement(s) or regulation(s) or “difference in emission(s)” or “value(s) of difference” <p>DO NOT ACCEPT other similar keywords Ex: “excess” or “reduction”</p> <p>ACCEPT multiple keywords in one sentence and ignore additional irrelevant factors</p>		

Key

DC: The missing values in the row of Difference in carbon emissions

VC: The missing values in the row of Value of difference in carbon emissions **ACCEPT** negative values for buying (Ex: -0.48,-0.63,-0.72)

Year	2022	2023	2024	2025	2026	2027	2028	2029	2030
Difference in carbon emissions (kt)	28	19.5	12	5.5	0	4.5	8	10.5	12
Value of difference in carbon emissions (\$ million)	1.68	1.17	0.72	0.33		0.27	0.48	0.63	0.72
Buy or sell	sell	sell	sell	sell		buy	buy	buy	buy

Total of Sell is: 1.68+1.17+0.72+0.33=3.9

Total of Buy is: 0.27+0.48+0.63+0.72=2.1

Mark	1	2	3	4
Calculate (C)	<p>One from: (see table above)</p> <ul style="list-style-type: none"> Correctly write three DC Correctly calculate four of <u>their</u> VC Correctly calculate cost if no filters installed: $(222 \times 60k) = 13.32$ (\$ mil) ACCEPT - 13.32 Weak attempt to calculate <u>their</u> cost if filters installed: They forget the cost of filters or income from selling or price of buying <p>Ex: their 3.9 - their 2.1 or their 1.8 Ex: 14 - their 3.9 or their 10.1 Ex: 14 + their 2.1 or their 16.1</p>	<p>Two from: (see table above)</p> <ul style="list-style-type: none"> Correctly write three DC Correctly calculate four of <u>their</u> VC Correctly calculate cost if no filters installed: $(222 \times 60k) = 13.32$ (\$ mil) ACCEPT - 13.32 Good attempt to calculate their cost if filters installed: They include cost of filters, income from selling, and price of buying but incorrect final result <p>Ex: 14 + their 3.9 - their 2.1 or their 15.8 (\$mil)</p> <p>ACCEPT <u>the</u> correct costs without final result Ex: filters 14 then sell for 3.9 then buy for 2.1 WTTE</p>	<p>Three from: (see table above)</p> <ul style="list-style-type: none"> Correctly write all DC Correctly calculate all of <u>their</u> VC Correctly calculate cost if no filters installed: $(222 \times 60k) = 13.32$ (\$ mil) ACCEPT - 13.32 Correctly calculate <u>their</u> cost if filters installed <p>$(14 - \text{their } 3.9 + \text{their } 2.1 =) \text{their } 12.2$</p> <p>ACCEPT - their 12.2 (\$mil)</p>	<p>The four: (see table above)</p> <ul style="list-style-type: none"> Correctly write all DC Correctly calculate all of <u>the</u> VC Correctly calculate cost if no filters installed: $(222 \times 60k) = 13.32$ (\$ mil) ACCEPT - 13.32 Correctly calculate <u>the</u> cost if filters installed: <p>$(14 - 3.9 + 2.1 =) 12.2$ (\$mil)</p> <p>ACCEPT - 12.2 (\$mil)</p>

Mark	1	2
Recommendation (J)	<p>ACCEPT only if C1 is achieved</p> <p>Recommendation with one weak justification considering the 8 years period or beyond the 8 years period</p> <p style="text-align: center;">Examples considering the 8 years period</p> <p>Ex: Filters will save money WTTE (and we see values aligned with this recommendation even if incorrectly calculated)</p> <p>Ex: Should install filters but in 4 years they will have to buy new ones WTTE</p> <p>Ex: Should not install filters because after 2026 (or starting 2027) filters will not be efficient enough WTTE</p> <p>Ex: Should not install because it is not economic (and we see values aligned with this recommendation even if incorrectly calculated)</p> <p>Ex: should not install filters because the price of filters is paid upfront WTTE.</p> <hr/> <p style="text-align: center;">DO NOT ACCEPT seeing only price of filters too high</p> <hr/> <p style="text-align: center;">OR Examples considering beyond 8 years period</p> <p>Ex: After 2030 (or after 8 years), installing filters will be no good or they will have to buy new filters or take new measures WTTE</p> <p>Ex: On the long run (in the future) they will have to buy more filters WTTE</p> <hr/> <p style="text-align: center;">ACCEPT not recommending filters if their justification supports it</p> <hr/> <p style="text-align: center;">DO NOT ACCEPT "install filters because it is better for environment" WTTE</p>	<p>ACCEPT only if C4 is achieved</p> <p><u>Recommendation to install filters</u> with one good justification aligned with the correct calculations</p> <p>Ex: Should install filters because they will save 1.12 (mil\$) WTTE</p> <p>Ex: Should install filters because without filters they will spend 13.32(mil) while with filters only 12.2 WTTE</p> <p>ACCEPT Should install filters because it will save money WTTE, and we see 13.32(mil\$) and 12.2(mil\$) in the calculations</p>

Mark	1	2
Accuracy (A)	Implies inaccurate with one justification <ul style="list-style-type: none"> Price of buying/selling (60000) carbon units is approximate WTTE OR <u>Limits/restrictions</u> could change (based on world events or to decrease pollution or because climate change). WTTE 	Implies inaccurate with two justifications <ul style="list-style-type: none"> Price of buying/selling (60000) carbon units is approximate WTTE OR <u>Limits/restrictions</u> could change (based on world events or to decrease pollution or because climate change). WTTE
	OR <ul style="list-style-type: none"> <u>(Carbon) emissions</u> are predictions/estimates or might change over the years WTTE 	AND <ul style="list-style-type: none"> <u>(Carbon) emissions</u> are predictions/estimates or might change over the years WTTE
	<p style="text-align: center;">ACCEPT</p> “values used are estimations or approximated” WTTE for A1 <p style="text-align: center;">DO NOT ACCEPT</p> “inaccurate because I rounded” WTTE accurate regardless their justification just rounding their results	<p style="text-align: center;">DO NOT ACCEPT</p> “inaccurate because I rounded” WTTE accurate regardless their justification just rounding their results

Q7 Task 3

Glossary for task 3

Term used	Clarification
General rule	Rule in terms of only n (if they use x , it is still general rule but penalise in notation)
The general rule	The correct general rule in terms of only n (if they use x , it is still the general rule but penalise in notation)
Their general rule	valid attempt for the general rule but in terms of only n (if they use x , it is still their general rule but penalise in notation)
Their rule	Correct rule not in terms of only n
Recursive rule	$A = \frac{A_{n-1}}{4} OE$

We accept subsequent use of their general rule provided it is of similar complexity. In general, the complexity of the rule depends on its form. The table below shows examples.

The general rule	ACCEPT for their general rule	DO NOT ACCEPT for their general rule
$T = 8n - 8$	Linear	
$A = \frac{8}{2^n}$	Quadratic or exponential	Linear

7	Answers	Notes	Total
a	Correctly place 32 and 40	5	32
		6	40
			1

<p>b</p>	<p>.1 correctly describe one pattern for T in words</p> <p>.2 correctly describe a second pattern for T in words</p>	<p>ACCEPT complete terminology only, for example (below are four different descriptions)</p> <p>divisible by 8, multiples of 8 Increasing by 8, adds 8, goes up by 8, moving up by 8 difference 8, common difference 8, linear with difference 8, arithmetic with difference 8 Second difference zero</p> <p>DO NOT ACCEPT incomplete terminology, for example: Arithmetic, linear, increasing by a constant, constant difference</p> <p>DO NOT ACCEPT general description, for example: Integers, whole numbers, positive, even numbers, divisible by 2, multiples of 2, multiples of 4</p> <p>DO NOT ACCEPT the rule in words or description related to n for example: 8 times n minus 8, n multiplied by 8 minus 8, WTTE</p> <p>Note, in the case when they have more than two different patterns:</p> <p>If two are accepted and the rest are all correct: award 2 marks Ex: even numbers and adds 8 and Second difference is zero and it is 8 times n then subtract 8. Ex: multiples of 2 and 4 and linear and increases by 8 and divisible by 8</p> <p>If two are accepted and any of the rest is incorrect: award 1 mark Ex: <u>increases by 2</u> and Second difference is zero and it is divisible by 8</p> <p>If only one is accepted, ignore the rest and award 1 mark</p>	<p>2</p>
<p>c</p>	<p>.1 the correct general rule</p> <p>.2 the correct simplified general rule with correct notation for T in terms of n.</p>	<p>.1 $8n - 8$ or $8(n - 1)$ or $T = 8 \times n - 8$ or $(T =) 8 * n - 8$ or $T = 8 \times (n - 1)$ or $T = 0 + 8(n - 1)$</p> <p>.2 $T = 8n - 8$ or $T = 8(n - 1)$</p> <p>.2 ACCEPT using Tn instead of T</p> <p>.2 ACCEPT $T = n8 - 8$ or $T = (n - 1)8$</p> <p>.2 ACCEPT using t and N</p> <p>DO NOT ACCEPT description in words</p>	<p>2</p>

<p>d</p>	<p>.1 correctly substitute $n \geq 5$ into their general rule (from 7c or 7b)</p> <p>.2 correctly calculate their value of T after substituting $n \geq 5$</p> <p>.3 recognise that their correctly calculated value of T is the same as their predicted value</p>	<p>.1 Ex: $8 \times 5 - 8$</p> <p>.2 Ex: 32 (for $n = 5$)</p> <p>.3 "the same as when we continue the pattern" WTTE and states how Ex: for $n=7$, 48 is obtained from pattern of adding 8 to 40 .3 ACCEPT if their value from .2 is the same as their value in the table in part a) or seen here in part d) Ex: we see their calculated $T=32$ and we see $T=32$ in their table</p>	<p>3</p>
<p>e</p>	<p>AM1 (rhombus) Correctly half the product of the two diagonals</p> <p>AM2 (rectangle – triangles) Correctly subtract area of 4 triangles from the rectangle</p> <p>AM3 (4 triangles) Correct area of 4 triangles or 2 big triangles that form 1 rhombus</p> <p>$\frac{1}{16}$ AG</p>	<p>AM1 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}$</p> <p>AM2 $\frac{1}{8} - \frac{1}{16}$ Comes from 4 triangles each being $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{8} = \frac{1}{64}$ and 1 rectangle $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$</p> <p>AM3 $4 \times \frac{1}{64}$ or $2 \times \frac{1}{32}$</p> <p>Comes from area of 4 triangles each being $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{8}$ or 2 triangles each being $\frac{1}{2} \times \frac{1}{8} \times \frac{1}{2}$ or $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}$</p> <p>ACCEPT notation errors</p> <p>ACCEPT lengths in decimals only if $\frac{1}{16} = 0.0625$ is seen</p> <p>Ex : $0.5 \times 0.5 \times 0.25 = 0.625$ AND $\frac{1}{16} = 0.0625$</p> <p>DO NOT ACCEPT $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}$ that comes from $A = b^2 \times h$</p> <p>DO NOT ACCEPT $\frac{1}{4} \times \frac{1}{4}$</p> <p>DO NOT ACCEPT working only with denominators Ex : denominators $2 \times 2 \times 4 = 16$</p>	<p>1</p>

Q7e 23 marks

Mark	1	2	3
Predictions (P) ACCEPT whether in the table or in the response box	Correctly predict two terms for L or W	Correctly predict two terms for L or W AND Correctly predict one term for A	Correctly predict two terms for L and two terms for W
	OR	OR	AND
	Correctly predict one term for A	Correctly predict two terms for A	Correctly predict two terms for A
	Ignore additional incorrect predictions ACCEPT fractions or index form ACCEPT in the table or in the response box DO NOT ACCEPT decimals	Ignore additional incorrect predictions ACCEPT fractions or index form ACCEPT in the table or in the response box DO NOT ACCEPT decimals	Ignore additional incorrect predictions ACCEPT fractions or index form ACCEPT in the table or in the response box DO NOT ACCEPT decimals

Stage number (n)	Length of big diagonal (L)	Length of small diagonal (W)	Area of rhombus (A)
5	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{256}$
6	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{1024}$
7	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{4096}$
8	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{16384}$

Examples of rules in equivalent forms (accept decimals) :

$$L = \frac{4}{2^n} = 2 \times \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^{n-2}} = 2^{2-n}$$

$$W = \frac{2}{2^n} = 1 \times \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^{n-1}} = 2^{1-n}$$

$$A = \frac{1}{4^{n-1}} \text{ or } A = \frac{1}{2^{2n-2}} \text{ or } A = 4^{1-n} \text{ or } A = 2^{2-2n}$$

Mark	1	2	3	4	5	
Description (D) ACCEPT notation errors but penalized in notation (N)	Correctly describe a pattern in words for A Ex: Multiply by $\frac{1}{4}$ OE, Divide by 4, Divide by 2 two times (Geometric with) ratio $\frac{1}{4}$ OE. Denominators multiply by 4 or they are divisible by 4 or they all have 4 as a factor or they are all powers of 4. Denominators <u>except first</u> are multiples of 4.	Correctly describe a pattern in words for A AND Correct recursive rule for A or rule $A=4^n$ or Valid attempt to write down general rule for L or W	Correctly describe a pattern in words or recursive rule for A AND Valid attempt to write down a general rule for A (at least seeing $(an+b)$ as an exponent)	Correctly describe a pattern in words or recursive rule for A AND Correctly write down the general rule for L or W	Correctly describe a pattern in words or recursive rule for A AND Correctly write down the general rule for A	
	OR					
	Recursive rule for A $A = \frac{A_{n-1}}{4}$ OE or the rule $A=4^n$					
	OR					
	Valid attempt to write down a general rule for L or W (at least seeing n as an exponent) ACCEPT if they correctly describe a pattern in words or recursive rule for L or W Ex: Multiply by $\frac{1}{2}$ OE, Divide by 2 (Geometric with) ratio $\frac{1}{2}$ OE Denominators are multiples of 2 or divisible by 2 or even numbers or powers of 2 $L = \frac{L_{n-1}}{2}$ or $W = \frac{W_{n-1}}{2}$ OE	OR Valid attempt to write down a general rule for A (may be incorrect but contains at least $(an+b)$ as an exponent)	OR Correctly write down the general rule for L or W $L = \frac{4}{2^n}$ OE $W = \frac{2}{2^n}$ OE	OR Correctly write down the general rule for A $A = \frac{1}{4^{n-1}}$ OE		
	IGNORE additional incorrect patterns					
	DO NOT ACCEPT					
	Exponential, geometric, arithmetic sequence, denominators are multiples of 2 or even numbers, the square numbers, A is decreasing $A = \frac{1}{2}(L \times W)$ The rule for A in words as a pattern.					

Mark	1	2	3
<p>Testing (T)</p> <p>ACCEPT transforming into decimals when testing</p>	<p>Attempt to test their general rule for A using $n \leq 4$</p> <p>Ex: Substitute in their general rule for A value of $n \leq 4$</p> <p style="text-align: center;">OR</p> <p>Correctly test their described pattern or their rule (e.g. recursive rule) or their rule for L or W in terms of n</p> <p style="text-align: center;">DO NOT ACCEPT testing their rule of $A=0.5xLxW$</p>	<p>Correctly test their general rule for A, using $n \leq 4$</p> <p>Ex: Correctly calculate their value for A in their general rule using $n \leq 4$</p> <p style="text-align: center;">AND</p> <p>Recognise that their correctly calculated value for A is the same as the given value.</p> <p>ACCEPT seeing their correctly calculated value for A and the given value in the table being equal</p>	
<p>Verifying (V)</p> <p>ACCEPT transforming into decimals when verifying</p>	<p>Attempt to verify their general rule for A using $n \geq 5$</p> <p>Ex: Correctly substitute in their general rule for A value of $n \geq 5$</p> <p style="text-align: center;">OR</p> <p>Correctly verify their described pattern or their rule (e.g. recursive rule) or their rule for L or W in terms of n</p> <p style="text-align: center;">DO NOT ACCEPT verifying their rule of $A=0.5xLxW$</p>	<p>Correctly calculate their value for A in their general rule, using $n \geq 5$</p>	<p>Correctly calculate their value for A in <u>the general rule</u> using $n \geq 5$</p> <p style="text-align: center;">AND</p> <p>Recognise that their correctly calculated value for A is the same as the correct predicted value obtained by continuing the pattern</p> <p>ACCEPT seeing their correctly calculated value for A and the correctly predicted value in the table being equal</p>

Mark	1	2	3	4
Justify (J)	ACCEPT only if D1 is achieved Attempt to use geometric sequence to justify <u>their</u> general rule for A or described pattern or rule or recursive rule Ex : Every time we multiply by $\frac{1}{4}$ so it makes sense to have 4 in my general rule WTTE and we see 4 or $\frac{1}{4}$ in <u>their</u> general rule for A	ACCEPT only if D4 is achieved Correctly use geometric sequence to justify <u>the</u> general rule aligned with their notation for <u>the</u> general rule for A Ex : Assume $a \times r^n$ and substitute $r = \frac{1}{4}$ and a value for A and n then find $a = 4$	ACCEPT only if D4 is achieved Attempt to justify geometrically the general rule for A Half of the product of correct L and W rules without complete simplification Seeing $\frac{L \times W}{2} = \frac{4 \times 2}{2 \times 2^n \times 2^n} \text{ OE}$ or $\frac{4 \times 2}{2 \times 2^n \times 2^n} = \frac{4}{4^n} \text{ OE}$	ACCEPT only if D4 is achieved Correctly justify geometrically the general rule for A Half the product of correct L and W with simplification. ACCEPT simplification using factor 4 instead of 2. They show how it simplifies to $A = \frac{1}{4^{n-1}} \text{ or } A = 4^{1-n}$ $A = \frac{1}{4^{n-1}} \text{ or } A = \frac{1}{2^{2n-2}} \text{ or } A = 4^{1-n} \text{ or } A = 2^{2-2n}$
	OR using parameters Assume $a \times r^n$ and substitute $r = \frac{1}{4}$ and a value for A and n then find incorrect a	OR Weak attempt to justify <u>the</u> general rule for A geometrically by using half the product of incorrect L and W rules or by using correct L and W rules but incorrectly halved or with mistakes in simplification		
	OR Weak attempt to justify <u>their</u> general rule for A geometrically by using the product of <u>incorrect</u> L and W <u>rules</u> OR L is halved and W is halved so A is divide 4 WTTE			
	OR Substitute at least two other values of n in their general rule for A, L or W and say they are the same or the rule works WTTE			
	OR Recognize it is geometric progression and define first term 1 and ratio $\frac{1}{4}$ WTTE			
DO NOT ACCEPT A is square of W				

Mark	1	2	3
Notation and terminology (N)	<p>CORRECT NOTATION</p> <ul style="list-style-type: none"> - using U_n instead of A only if they mention that $A = U_n$ - Using decimals <p>NOTATION WITH ERRORS</p> <ul style="list-style-type: none"> - using * for multiplication - using / for division - using ^ for power - using x instead of n - “the rule for A is:” instead of “A=” 		
	<p>ACCEPT only if D1 is achieved</p>	<p>ACCEPT only if D4 is achieved</p>	<p>ACCEPT only if D5 is achieved</p>
	<p>Correct notation of <u>their</u> general rule for A or rule for L or W</p> <p>Ex:</p> $A = 4^n \quad \text{or} \quad L = \frac{4}{2^n} \quad \text{or} \quad W = \frac{2}{2^n}$	<p>Correct notation of <u>the general</u> rule for A in simplest form (see examples below and above)</p> <p>ACCEPT simplification using factor 4 instead of 2.</p> <p>Ex: $A = \frac{1}{4^{n-1}}$ or $A = \frac{1}{2^{2n-2}}$</p> <p>or $A = 4^{1-n}$ or $A = 2^{2-2n}$</p> <p>And OE for decimals</p> <p>Ex: $A = 0.25^{n-1}$ or $A = 0.5^{2n-2}$</p>	<p>Correct notation of <u>the general</u> rule for A in simplest form (see examples in N2 and above)</p> <p style="text-align: center;">AND</p> <p>Correctly describe a pattern in words for A</p>
	<p style="text-align: center;">OR</p> <p>The notation of <u>the general</u> rule includes errors or non-simplified or in words</p> <p>Ex:</p> $A = 1/4^{(n-1)} \quad \text{or} \quad A = 4/4^n \quad \text{or} \quad A = \frac{4}{4^n}$ <p>or Missing the “A=”</p> <p>Ex: The rule for A is $\frac{1}{4^{n-1}}$</p>	<p style="text-align: center;">OR</p> <p>The notation of <u>the general</u> rule includes errors or not simplified or in words (see examples in N1 and above)</p> <p style="text-align: center;">AND</p> <p>Correctly describe a pattern in words for A</p>	
<p style="text-align: center;">OR</p> <p>Correctly describe a pattern or recursive rule in words for A</p> <p>Ex: $A = \frac{A_{n-1}}{4}$</p>			

<p>Communication (L)</p> <p>Organisation and coherence can be awarded even if there are errors</p> <p>For items: Describing pattern and writing rule can be considered an item even if D0 awarded Test, verify, and justify may include errors but cannot be considered item (or identified for coherence) if they are zeros</p>	<p>At least three of the following items are seen:</p> <ul style="list-style-type: none"> - describe a pattern in words - write a rule - test their general rule or rule or recursive rule or pattern (at least T1) - verify their general rule or rule or recursive rule or pattern (at least V1) - justify their general rule or rule or recursive rule or pattern (at least J1) 	<p>At least four of the following items are seen:</p> <ul style="list-style-type: none"> - describe a pattern in words - write a general rule - test their general rule - verify their general rule - justify their general rule <p style="text-align: center;">AND</p> <p>For coherence, they identify the processes correctly. At least one from the following:</p> <ul style="list-style-type: none"> - test - verify - justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in their general rule value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in their general rule value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in their general rule value(s) of $n \leq 4$ followed by value(s) of $n \geq 5$, consider it as only one identified process</p>	<p>ACCEPT only if D4 and J2 are achieved</p> <p>At least four of the following are seen:</p> <ul style="list-style-type: none"> - describe a pattern or rule in words - write <u>the general rule</u> - test <u>the general rule</u> - verify <u>the general rule</u> - justify <u>the general rule</u> <p style="text-align: center;">AND</p> <p>For coherence, they identify the processes correctly. At least two from the following:</p> <ul style="list-style-type: none"> · test · verify · justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in their general rule value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in their general rule value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in their general rule value(s) of $n \leq 4$ followed by value(s) of $n \geq 5$, consider it as only one identified process</p>
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